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THE CIVIL ENGINEER.

A Prospect.

BY

FRANK M. OKEY.

Presumably because he is a teacher of Civil Engineering, the writer frequently finds it necessary to answer the question, "What does the future hold for the Civil Engineer?" The query comes most often from college students, but not infrequently from those outside of college. Each time the answer has been the same and given with an ever strengthening conviction, that the prospects for the Civil Engineer have not been brighter for fifteen or twenty years; that the promises of his future are pleasant indeed.

In fact, all signs seem to indicate that the Civil Engineer is coming to his own again. "Again," because for some years past, many a competent man has been hard pressed to find steady employment, and even then at a remuneration far less than he deserved or earned. There seemed to be two men for every position, especially the more subordinate ones, and employers of Civil Engineers took advantage of this fact by making salaries small and advances few and far apart. This condition was due, no doubt, to the fact that technical schools were graduating large classes from their Civil Engineering Departments, and also to the fact that there was some falling off in the amount of engineering work going on.

About a decade and a half, or more, ago, the great amount of public and private construction work actively in progress or being prospected, called for the services of large numbers of Civil Engineers, and so strong was the demand for men of this profession, that technical schools and colleges began featuring their Civil Engineering courses in their advertising campaigns. As a result, it almost became a fad to take civil engineering, and many of the schools actually boasted of the large numbers of "Civil Engineers" they were turning out.

As a matter of fact, many of these men were not of the stuff from which civil engineers are made. To the writer's certain knowledge, many of them never progressed farther than mediocre chainmen or indifferent instrument men even after several years' trying, and finally became discouraged and started clerking in grocery stores or drifted back to the farm; lines of work they should have taken up to begin with.

Though there was really something of a shortage of competent men for the more important positions, the profession seemed crowded, and many students just entering college were discouraged from taking the civil engineering work. This falling off was strikingly illustrated in one of the larger state universities of the middle west during the year 1911-12. The university graduated 72 men from the civil engineering department in 1912, while the number of freshmen enrolled in that department the same year was only 68. The writer has not statistics giving the number of C. E. graduates from this institution in 1915, but it is certainly safe to say that not more than 60% or about 41 men reached the senior year, and perhaps the graduates numbered even less than this. This same university graduated but 37 C. E.'s in 1917.

During the 22 years from 1895 to 1916, 11,621 Civil Engineers were graduated in America, and the rate of increase each year is certainly not upward. In a nation of a hundred million people, 11,621 civil engineers ought not to cause a *serious* overcrowding.

Of late years, students enrolling in civil engineering courses and staying through the hard grind to the completion of their course, are men with a natural aptitude for the profession. They are in the game to win, and about four out of every five are achieving enviable positions, and the fifth man bids fair to be reasonably successful.

The opportunities of the profession are increasing every year, and because of the increasingly small number who are entering the field annually, it is becoming more and more difficult to find men enough to fill all the positions. Moreover, because of the many would-be-civil-engineers who became grocery clerks or agriculturalists, it is difficult indeed to secure men of sufficient experience to fill the more important places. In fact the situation is becoming exceedingly grave.

The public mind has just begun to grasp the importance and broadness of the civil engineering profession and it is deplorable that more young men are not turning to this branch. New fields are being constantly opened up to the civil engineer or are being opened up by him. Like the Germanic hordes that overran southern Europe in the Fifth century, because they sought more lands to conquer, our civil engineering graduates are overflowing the bounds of their profession—though in no barbaric fashion—and forcing their way into industries that have never known scientific direction or control.

Agriculture is feeling the benefit of civil engineering energies directed toward it. Swampy lands have been drained and made to yield great returns. Deserts have had water applied to their surfaces and the results have been astounding. But there are yet no less than 14,000,000 acres of land in the north central states and many millions more in the southern states that are unproductive because of excess of water. Many of the tracts are real swamps, but much of this vast acreage is merely saturated with water. As it stands, this land has only a prospective value. Yet most of it can be drained at a cost far below the thus created value. Capital must be put into the land, but, before capital will come, brains must map the projects and disclose the profits. What sort of brains is more capable of reclaiming this acreage than the brains of the civil engineer promoters? No time is more opportune than the immediate future for the promotion of land reclamation.

Opposed to the swamps are the millions of acres still without water to produce vegetation, and here again the civil engineer faces the open door of opportunity. Though more than 16,000,000 acres have been made productive through his efforts, there still remains far more millions that must be made to produce if our nation would continue to hold its place as one of the world's greatest granaries.

With the ever-increasing use of the automobile, there comes an ever-increasing demand for better and more extensive systems of public highways. The various states seem to be vieing with each other as to which will lead in the number of miles of improved roads added to its total each year, and the work is but little more than well started. In this field the civil engineer is finding greater and greater demands for his services and there seems to be no prospect of a slowing up for many years.

The small army of designers in reinforced concrete and steel is growing yearly, and there is no prospect whatever that recruits to its ranks will not be more than welcome for many years to come. Steel long ago proved its right to the claim of being a superior material of construction, and reinforced concrete is widening its field of usefulness and proving itself more dependable every year.

A great deal is heard these days of the "City Manager." He is a fact in many of the middle-sized cities of this country, and it is interesting indeed to note that an ever-increasing number of city managers are being drawn from the ranks of the civil engineer. Now comes the "*County* Manager," and it is worthy of note that the first of this new profession is a civil engineer. A study of the problems confronting these city and county "Managers" makes it very evident that the civil engineer is pre-eminently the man who is properly fitted to handle the position.

Municipalities all over the country are needing more and more civil engineers for design and superintendence of construction and for more efficient maintenance work. The editor of "Engineering and Contracting," a leading technical journal published in Chicago, says: "It is probably not overestimating matters to say that fully 50,000 civil engineers are needed, and badly needed, to manage municipalities and design and supervise city work in America. * * * Although we have barely made a start along the line of executive engineering, it is patent that the public is in a mood for a change from old-time political to new-time engineering management of city affairs." Yet with all this need, only about 12,000 civil engineers under the age of 45 are available. There does not seem to be much overcrowding in this direction.

Valuation work, both Federal and for private corporations, has called hundreds of civil engineers during the last few years, and this field is ever broadening.

It is predicted that civil engineers will shortly be heard from in politics—even now they are silently entering that field—and then we may hope for that combined honesty and intelligence of administration so sadly lacking now in our municipal governments. Indeed, municipal government's greatest concern today is the economic building and maintaining of civil engineering structures, such as water works, sewers, pavements, coupled with the regulation of public service corporations (also of an engineering nature) such as street railways, gas and electric plants.

The foregoing has outlined the civil engineering future only from a peace standpoint. Without a war, with its great problems of reconstruction, peaceful America alone was bright with future opportunities. On the other side of the Atlantic is a civilization sadly shattered if not totally paralyzed by a stupendous war, and no one knows when it will end. But it must end some day, as all wars have ended, and then will present themselves, problems of reconstruction of a vastness never before known. The wheels of industry must be started turning again. Railroads must be built and a demoralized traffic returned to a rational course, not only in Europe, but in the United States, where every energy is being centered on problems of transportation while reconstruction and maintenance are largely allowed to shift for themselves. Ships, factories and even cities must be built, here and abroad. Mines must be opened. The conditions of a war furrowed world must be met.

The engineers of America and pre-eminently the civil engineers, must take a leading part in this vast reconstruction work to come. The countries of Europe, which in days past have produced such capable engineers, will be able to help in this respect but slightly. Thousands of the keenest and most capable engineers have been sacrificed on the battle fields, and thousands more are incapacitated forever because of wounds and sickness. Even our own engineers, reluctant though we may be to consider the prospect, will some of them meet death and many more return at the end of the war with their efficiency entirely gone. So the men who are left or who are advanced to the profession in the next few years, will have even more duties before them.

Our war torn neighbor to the south, opens the door to the civil engineer, and there are problems of reconstruction and development that will be years in the solving. Most of the engineering work done in Mexico before the days of the revolution was done by American engineers, and there is no other country from which they can expect to get help for a long time.

So, all things considered, it is apparent that the prospect of the civil engineer is promising indeed. As a matter of fact, it is more than evident that there will be a serious shortage of trained men for years to come. The war needs of our own country have lowered our number of available engineers and caused a serious reduction in enrollment in the various schools and colleges. Many European technical schools have closed their doors or have but a very few students. America must meet the world's demand for technically trained men, many of whom will be civil engineers. When we consider the great diversity of fields calling the technically trained civil engineer, it is apparent at once that unparalleled opportunities are offered to the young man fitted by nature and temperment to embrace them.

NEWTON'S SOLUTION OF NUMERICAL EQUATIONS BY MEANS OF SLIDE RULES.

BY

FLORIAN CAJORI.

Inasmuch as everything pertaining to Sir Isaac Newton is of general interest to scientific men, it seems worth while to call attention to his use of logarithmic slide rules in securing rough approximations to the real roots of numerical equations. Well-known to all who are familiar with the history of the slide rule is the letter, dated June 24, 1675, which Oldenburg, the then Secretary of the Royal Society of London, addressed to Gottfried Wilhelm Leibniz. That letter containing four sentences on this use of slide rules, is given in the collected works of Newton (*Opera,* Edition S. Horsley, Vol. 4, London, 1782, p. 520) and is referred to in the *Encyclopédie des sciences Mathématiques,* Tome I, Vol. 4, page 429.

A second fuller account of Newton's procedure is described by E. Stone in his *New Mathematical Dictionary,* second edition, London, 1743, in an article at the end of the volume, which, as Stone says, is "To be added to the Head of Roots of Equations." In this article Stone gives a description of Newton's process, which has been made more easily accessible to modern readers, by a re-print of it in Cajori's *History of the Logarithmic Slide Rule,* New York, 1909, pages 26-28.

Only recently the present writer accidentally discovered a third account of Newton's process, which purports to be taken directly from Newtonian manuscripts and is printed in James Wilson's *Mathematical Tracts of the late Benjamin Robins,* London, 1761. This book is out of print and is not easily procurable through second hand dealers. It is practically inaccessible to the great mass of scientific workers. Being the most satisfactory account of the three which we have named, it may be of some interest to reproduce it, especially as it is not included in Newton's collected works. For the sake of completeness we reprint all three accounts, so that they may easily be compared with each other. We remark, in passing, that there are still manuscripts of Newton which have never been minutely examined and have not been published. Whether the manuscript from which James Wilson took the account printed in his book is extant or not, we have not yet been able to ascertain.

I. *Extract from Oldenburg's letter to Leibniz, June 24, 1675.*

We translate the Latin passages containing this description as follows:

"Mr. Newton, with the help of logarithms graduated upon scales by placing them parallel at equal distances or with the help of concentric circles graduated in the same way, finds the roots of equations. Three rules suffice for cubics, four for biquadratics. In the arrangement of these rules, all the respective coefficients lie in the same straight line. From a point of which line, as far removed from the first rule as the graduated scales are from one another, in turn, a straight line is drawn over them, so as to agree with the conditions conforming with the nature of the equation; in one of these rules is given the pure power of the required root."

II. *From James Wilson's Mathematical Tracts of the late Benjamin Robins, 1761, Vol. II, Appendix, pages 346-530.*

"About the year 1708, all the papers of Mr. John Collins, who had kept a correspondence with the most eminent geometers in Europe, fell into the possession of Mr. William Jones, then a teacher of the mathematicks in this city. Amongst them were copies of some treatises of Sir Isaac Newton, particularly his Analysis per Aequationes numero terminorum infinitas.* These Mr. Jones communicated to their author, who thereupon lent him the abovementioned tracts which Mr. Jones transcribed, and used to distribute fragments of them to his scholars. So that many such are in the hands of different persons.

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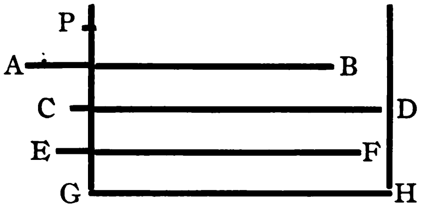
*Phil. Trans. No. 342, p. 176, or Com. Epist. p. 5.

"Amongst them was a small tract divided into two parts. In the first leaf was written 'Constructiones Geometricae Aequationum per D. Isaac Newtonum. Ex Apographo D ni Collins.' . . . As some of these constructions are particularly accommodated for obtaining the first figures of the roots of equations;* so Sir Isaac Newton found out two other expedients for the same purpose. The first of these he takes notice of in the abovementioned letter to Mr. Collins, and it was described by Mr. Oldenburgh in a letter to M. Leibnitz, dated the 24th of June, 1675.† But I shall here give a particular account of both.

"If within a frame several equal lines of logarithmick numbers, such as are described on Gunter's scale, AB, CD, EF and GH are placed equidistant from one another, and all but the last GH upon separate moveable sliders, and upon the fixt side of the frame a point P be placed at the same distance from the first of those lines. Then, when the beginning of all these lines stand directly under the Point P, a ruler laid from P to any number upon the first line AB shall mark upon the second line CD the square of that number, upon the line EF the cube, and upon GH the fourth power of the same. But if any one of the sliders be moved backwards, the ruler shall mark upon that slider the respective power of the forementioned number multiplied by the number, which in this slider in its present situation stands directly under the point P.



"Therefore any equation $z^4 - az^3 - bz^2 + cz = m$ being given, if the slider EF be moved back, till in that slider the number *a*

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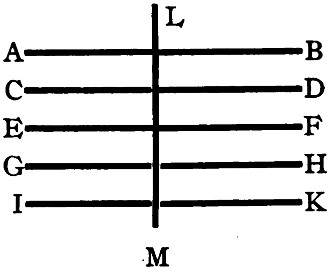
*Newton *Arith. Univers.* p. 285.

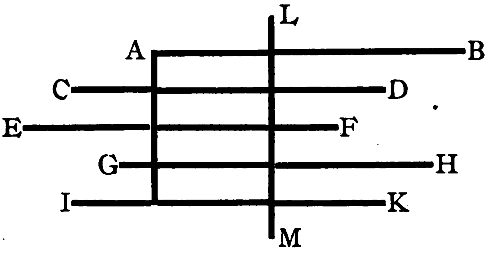
†"Dominus Newtonus, beneficio logarithmorum . . . quaesitae."

stand directly under the point P, and if the number *b* in the slider CD, and the number *c* in the slider AB, be likewise brought under P, then, if the ruler laid from P mark upon GH the true value of z^4, it will mark upon EF the value of az^3, upon CD the value of bz^2, and upon AB the value of cz, and these collected together under their proper signs will be equal to *m;* but when the numbers markt out by any situation of this ruler, being thus collected together, are either greater or less than *m,* then the ruler does not mark upon the line GH any true value of z^4, but by moving its situation, and collecting the numbers markt upon the sliders, all the values of z^4, and thence all the roots of the equation, may by repeated trials be found.

"His second method, which is more commodious, was thus. If within a frame several lines of numbers are placed after this manner. Suppose the first AB fixt upon the side of the frame, but the rest upon parallel sliders, the first of which CD shall be the same with AB, the second EF shall have its divisions but half as long, the divisions of the third GH one-third, and the divisions of the fourth IK one-fourth, of the divisions of the first; then, when the beginning of these lines stand under each other, if any line LM be drawn perpendicularly cross them, it shall mark upon the first of the sliders the same number as on the fixt line, upon the second slider the square of that number, upon the third the cube, and upon the fourth the fourth power of the same. But if any one of these sliders be moved backwards, then shall the transverse line LM mark upon that slider the respective power of the forementioned number multiplied by the number, which in this slider, in its present situation, stands directly under the point A, the beginning of the fixt line.



"Now let any equation be given $az + bz^2 - cz^3 + dz^4 = m$. If the slider CD be moved back, till in that slider the number a stand directly under A, and if the number b in the slider EF, and the number c in the slider GH, and the number d in the slider IK, are also brought under A; then, if the transverse line LM mark upon the line AB the true value of z, the root of the equation, it will give upon CD the value of az, upon EF the value of bz^2, upon GH the value of cz^3, and upon IK the value of dz^4; insomuch that the numbers markt upon the sliders by the transverse line LM, collected together under their proper signs, shall be equal to m.



"But what Sir Isaac Newton had done by these sliding rules, may be effected by any single line of artificial numbers, though not so expediciously; for if AB be such a line, in which the number expressing the value of the root z of the foregoing equation be at C, the distance AC taken from the number a will give upon this line az, and double this distance taken from b will give bz^2, and triple the same distance taken from c will give cz^3, and four times the distance taken from d will give dz^4. Which indeed may be performed most commodiously by these numbers inscribed on a circle after Oughtred's manner."*

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*See his Treastise, called the Circles of Proportion, etc., printed at London in 1633.

III. *From E. Stone's New Mathematical Dictionary, Second Edition, London, 1743 (last article).*

"But to speak Truth, whatever Arithmetical Rules have hitherto, or ever will be given for finding the Roots of Equation of more than three Dimensions in finite Terms, must from the Nature of the Thing be not worth the Pains and Perplexity in computing them by reason of their unelegance and length; for ever encreasing with the Number of Dimensions of the Equation, whose Roots are to be sought. I shall only mention a way of Sir Isaac Newton's of finding the Roots of Numerical Equations by means of Gunter's Lines sliding by one another.

"Take as many Gunter's Lines, (upon narrow Rules) all of the same Length, sliding in Dove-tail Cavities, made in a broad oblong Piece of Wood, or Metal, as the Equation whose Roots you want the Dimensions of, having a Slider carrying a Thread or Hair backward or forwards at right Angles over all these Lines, and let these Gunter's consist of two single ones, and a double, triple, quadruple, etc., one fitted to them; that is, let there be a fixed single one a top, and the first sliding one next that, let be a single one, equal to it, each Number from 1 to10. Let the second sliding one be a double Line of Numbers, number'd 1, 2, 3, 4, 5, 6, 7, 8, 9, to 10, in the Middle, and from 1 in the Middle to 1, 2, 3, etc., to 10, at the End. Let the third sliding one be a triple Line of Numbers, numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, and again 2, 3, 4, etc., to 10, and again 2, 3, 4, etc., to 100 at the End. The Distance from 1 to 1, 1 to 10, and 10 to 100, being the same; let the fourth sliding one, be numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 1; and again 2, 3, 4, etc., to 10; and again 2, 3, 4, etc., to 100; and again 2, 3, 4, etc., 1000. The Distance from 1 to 1, 1 to 10, 10 to 100, and 100 to 1000, being the same, and so on.

"This being done, take the Coefficient prefixed to the single Value of the unknown Quantity upon the fixed single Line of Numbers; the Coefficient of the Square of the unknown Quantity, upon the double Line of Numbers; the Coefficient of the Cube of the unknown Quantity, upon the triple Line of Numbers; the Coefficient of the Biquadrate of the unknown Quantity, upon the Quadruple Line of Numbers; and so on. And the Coefficient of the first or highest Term (being always Unity) take upon that Line of Numbers expressed by its Dimension, that is, if a Square, upon the double Line; a Cube, upon the triple Line, etc. I say, when this is done, slide all these Lines of Numbers so, that these Coefficients be all in a right Line directly over one another, and keeping the Rulers in this Situation, slide the Thread or Hair in such manner, that the Sum of all the Numbers upon the fixed single Line, the double Line, the triple Line, etc., which the Thread or Hair cuts, be equal to the Known Term of the Equation, which may be readily enough done with a little practice; and then the Number under the Thread upon that Line of Numbers of the same name with the highest Power of the unknown Quantity of the Equation, will be the pure Power of the unknown Quantity, whose Root may be had by bringing Unity on the single Sliding-Line directly over Unity upon this Line. After this, if you divide the Equation by this Root, you will have another, one Dimension less; and thus you may proceed to find a Root of this last Equation; which done, if it be divided by this last Root, you will get an Equation two Dimensions less, and by a Repetition of the Operation you will get a third Root, and so a fourth, fifth, etc., if the given Equation has so many, and if any of the intermediate Terms are wanting, the Gunter's express'd by the Dimensions of those Terms, must be omitted.

"But this Method only gives the Roots of Equations the Signs of all the Terms whereof, except the known one, are Affirmative; that is, of such that have all Negative Roots, but one, which last, the said Method finds. Therefore when an Equation is given, to find its Roots after this manner, whose Signs have other Dispositions, it must be first changed into another Equation, whose Signs are all Affirmative; but that of the known Term, which may be done by putting some unknown Quantity y Plus or Minus, some given Number or

Fraction, for the Value of the unknown Quantity x in the proposed **Equation.***

"Note, instead of streight Parallel Sliding Rules, you may have so many Gunter's Lines graduated upon Concentrick Circles, each moving under one another, by which Contrivance, you will have as large Divisions for your Logarithm within the Compass of one Foot, as you have upon a streight Ruler of more than three Feet in Length. Although perhaps by these Sliding-Rules, you cannot get all the Signs of the Roots exactly, for want of sufficient Subdivisions of the Gunter's Lines, yet if we can get two or three of the first Figures, it will be of good use to find the Roots by Approximation."

SUMMARY

The three accounts of Newton's processes of approximation to the real roots of numerical equations by the use of slide rules disclose four slightly different modes of procedure:

(1). The logarithmic rules are graduated alike, and are placed parallel and equidistant. Around a point as far removed from the first rule as the neighboring rules are from each other, a straight edge reaching across the rules is made to turn in the plane. This edge fulfills some of the functions of a runner. This arrangement is used in solving equations $x^n + ax^{n-1} + bx^{n-2} + \ldots + lx = m$, in which the coefficients $a, b, \ldots, l, m$ may be either positive or negative. It is explained in Oldenburg's letter and in Wilson's book.

(2). The rules are placed parallel and equidistant as in (1), but they are not alike; the first, second, third, etc., rules are graduated according to radii 1, ½, ⅓, etc. A "slider" or runner moves at right angles to the parallel rules. Equations may have positive or negative coefficients. Explained in Wilson's book.

(3). The arrangement explained in (2) is applied only

(**Note by F. Cajori:* The transformation of equations herein referred to is more fully discussed by Lagrange in Note XII of his Work, entitled *De la rèsolution des èquations numériques de tous les degrès*, the first edition of which appeared in Paris in the year VI (1798).)

to equations in which the coefficients $a, b, \ldots, l, m$ are all positive numbers. Explained in Stone's Dictionary.

(4). Concentric circles graduated logarithmically take the place of the parallel rules. Explained in Oldenburg's letter, Wilson's book and Stone's Dictionary.

An instrument analogous to that of Newton was designed for the solution of trinomial equations of any degree by the Spanish mathematician L. Torres. It is described by Maurice d'Ocagne in his *Traité de nomographie,* Paris, 1899, p. 366. Torres has invented several ingenious machines for the solution of equations. For references and explanations see also M. d'Ocagne's *Le calcul simplifié,* 2. Ed., Paris, 1905, pages 95, 123-128.